Project 1

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https://github.uio.no/FYS3150-G2-2023/Project-1

PROBLEM 1

First, we rearrange the equation.

$$
\begin{aligned}
-\frac{d^2u}{dx^2} &= 100e^{-10x} \\
\frac{d^2u}{dx^2} &= -100e^{-10x}
\end{aligned}
$$

Now we find $u(x)$.

$$
\begin{aligned}
u(x) &= \int\int \frac{d^2u}{dx^2} dx^2 \\
&= \int\int -100e^{-10x} dx^2 \\
&= \int \frac{-100e^{-10x}}{-10} + c_1 dx \\
&= \int 10e^{-10x} + c_1 dx \\
&= \frac{10e^{-10x}}{-10} + c_1 x + c_2 \\
&= -e^{-10x} + c_1 x + c_2
\end{aligned}
$$

Using the boundary conditions, we can find c_1 and c_2

$$
\begin{aligned}
u(0) &= 0 \\
-e^{-10 \cdot 0} + c_1 \cdot 0 + c_2 &= 0 \\
-1 + c_2 &= 0 \\
c_2 &= 1
\end{aligned}
$$

$$
\begin{aligned}
u(1) &= 0 \\
-e^{-10 \cdot 1} + c_1 \cdot 1 + c_2 &= 0 \\
-e^{-10} + c_1 + c_2 &= 0 \\
c_1 &= e^{-10} - c_2 \\
c_1 &= e^{-10} - 1
\end{aligned}
$$

Using the values that we found for c_1 and c_2, we get

$$
\begin{aligned}
u(x) &= -e^{-10x} + (e^{-10} - 1)x + 1 \\
&= 1 - (1 - e^{-10}) - e^{-10x}
\end{aligned}
$$

PROBLEM 2

PROBLEM 3

To derive the discretized version of the Poisson equation, we first need the Taylor expansion for $u(x)$ around x for $x+h$ and $x-h$.

$$u(x+h) = u(x) + u'(x)h + \frac{1}{2}u''(x)h^2 + \frac{1}{6}u'''(x)h^3 + \mathcal{O}(h^4)$$

$$u(x-h) = u(x) - u'(x)h + \frac{1}{2}u''(x)h^2 - \frac{1}{6}u'''(x)h^3 + \mathcal{O}(h^4)$$

If we add the equations above, we get this new equation:

$$\begin{aligned}
u(x+h) + u(x-h) &= 2u(x) + u''(x)h^2 + \mathcal{O}(h^4) \\
u(x+h) - 2u(x) + u(x-h) + \mathcal{O}(h^4) &= u''(x)h^2 \\
u''(x) &= \frac{u(x+h) - 2u(x) + u(x-h)}{h^2} + \mathcal{O}(h^2) \\
u_i''(x) &= \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + \mathcal{O}(h^2)
\end{aligned}$$

We can then replace $\frac{d^2u}{dx^2}$ with the RHS (right-hand side) of the equation:

$$\begin{aligned}
-\frac{d^2u}{dx^2} &= f(x) \\
\frac{-u_{i+1} + 2u_i - u_{i-1}}{h^2} + \mathcal{O}(h^2) &= f_i
\end{aligned}$$

And lastly, we leave out $\mathcal{O}(h^2)$ and change u_i to v_i to differentiate between the exact solution and the approximate solution, and get the discretized version of the equation:

$$\frac{-v_{i+1} + 2v_i - v_{i-1}}{h^2} = 100e^{-10x_i}$$

PROBLEM 4

PROBLEM 5

a)

b)

PROBLEM 6

a)

b)

PROBLEM 7

PROBLEM 8

PROBLEM 9