Project 2

Cory Alexander Balaton & Janita Ovidie Sandtrøen Willumsen
(Dated: September 25, 2023)

https://github.uio.no/FYS3150-G2-2023/Project-2

PROBLEM 1

We are studying the one-dimentional buckling beam, which can be described by the equation

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x) \qquad \rightarrow \qquad \frac{d^2 u(x)}{dx^2} = -\frac{F}{\gamma} u(x)$$

where γ is a constant determined by the material of the beam. We want to scale the equation, that is we want to scale by the x-value of the beams endpoint $x = L$. Scaling will result in a dimensionless variable $\hat{x} = \frac{1}{L}$.

$$\begin{aligned} \frac{d^2}{dx^2} &= \frac{d}{dx}\frac{d}{dx} = \left(\frac{d\hat{x}}{dx}\frac{d}{d\hat{x}}\right)\left(\frac{d\hat{x}}{dx}\frac{d}{d\hat{x}}\right) & \text{where we have used } \frac{d\hat{x}}{dx}\frac{d}{d\hat{x}} = \frac{d}{dx}\frac{d\hat{x}}{d\hat{x}} \\ &= \left(\frac{1}{L}\frac{d}{d\hat{x}}\right)\left(\frac{1}{L}\frac{d}{d\hat{x}}\right) = \frac{1}{L^2}\frac{d}{d\hat{x}^2} & \text{where } \hat{x} \equiv \frac{x}{L} \text{ and } \frac{d\hat{x}}{dx} = \frac{1}{L} \end{aligned}$$

Now we insert the expression into the original equation

$$\frac{du(\hat{x})}{d\hat{x}^2} = -\frac{FL^2}{\gamma} u(\hat{x})$$

PROBLEM 2

The functions that set up the tridiagonal matrices can be found in **matrix.hpp** and **matrix.cpp** in the Github repo.

The test for this can be found in **test_suite.cpp**.

PROBLEM 3

a)

The function to find the largest off-diagonal can be found in **matrix.hpp** and **matrix.cpp**.

b)

The test for (a) can be found in **test_suite.cpp**.

PROBLEM 4

a)

The code for Jacobi's rotation algorithm can be found in **jacobi.hpp** and **jacobi.cpp**.

b)

The test for (a) can be found in **test_suite.cpp**.

PROBLEM 5

a)

We used the Jacobi's rotation method to solve $\boldsymbol{A}\vec{v} = \lambda\vec{v}$, for $\boldsymbol{A}_{(N\times N)}$ with $N \in [5, 100]$, and increased the matrix size by 3 rows and columns for every new matrix generated. The number of similarity transformations performed for a tridiagonal matrix of is presented in Figure 1. We chose to run the program using dense matrices of same size as the tridiagonal matrices, to compare the scaling data. What we see is that the number of similarity transformations necessary to solve the system is proportional to the matrix size.

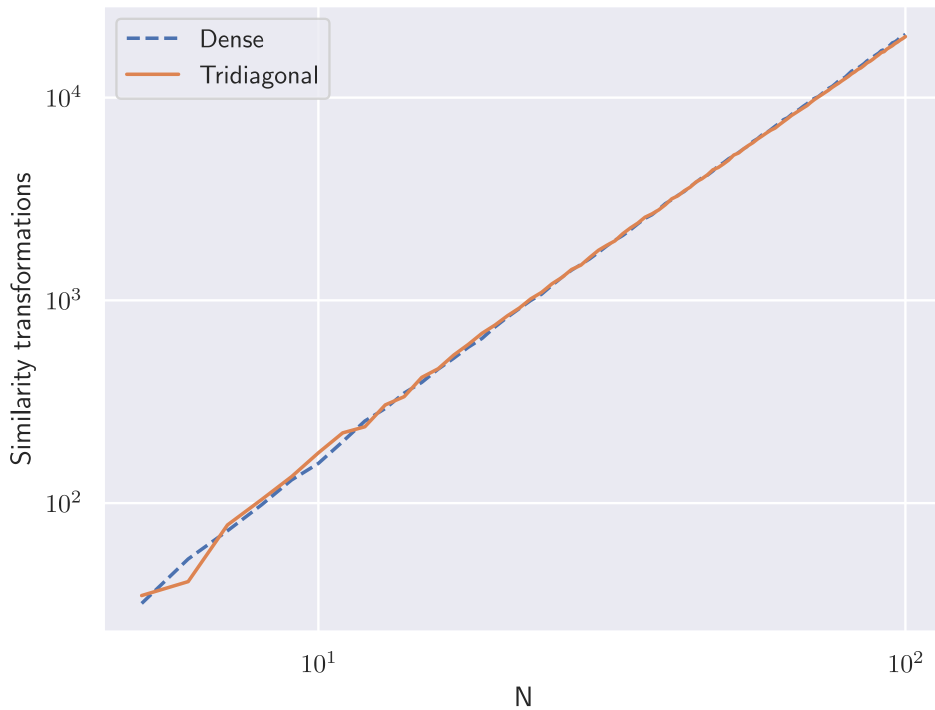

FIG. 1. Similarity transformations performed as a function of matrix size (N), data is presented in a logarithmic scale.

b)

For both the tridiagonal and dense matrices we are checking off-diagonal elements above the main diagonal, since these are symmetric matrices. The max value is found at index (k, l) and for every rotation of the matrix, we update the remaining elements along row k and l. This can lead to an increased value of off-diagonal elements, that previously were close to zero, and extra rotations has to be performed due to these elements. Which suggest that the number of similarity transformations perfomed on a matrix does not depend on its initial number of non-zero elements, making the Jacobi's rotation algorithm as computationally expensive for both dense and tridiagonal matrices of size $N \times N$.

PROBLEM 6

a)

The plot in Figure 2 is showing the discretization of $\hat{x}$ with $n = 6$. The eigenvectors and corresponding analytical eigenvectors have a complete overlap suggesting the implementation of the algorithm is correct. We have included the boundary points for each vector to show a complete solution.

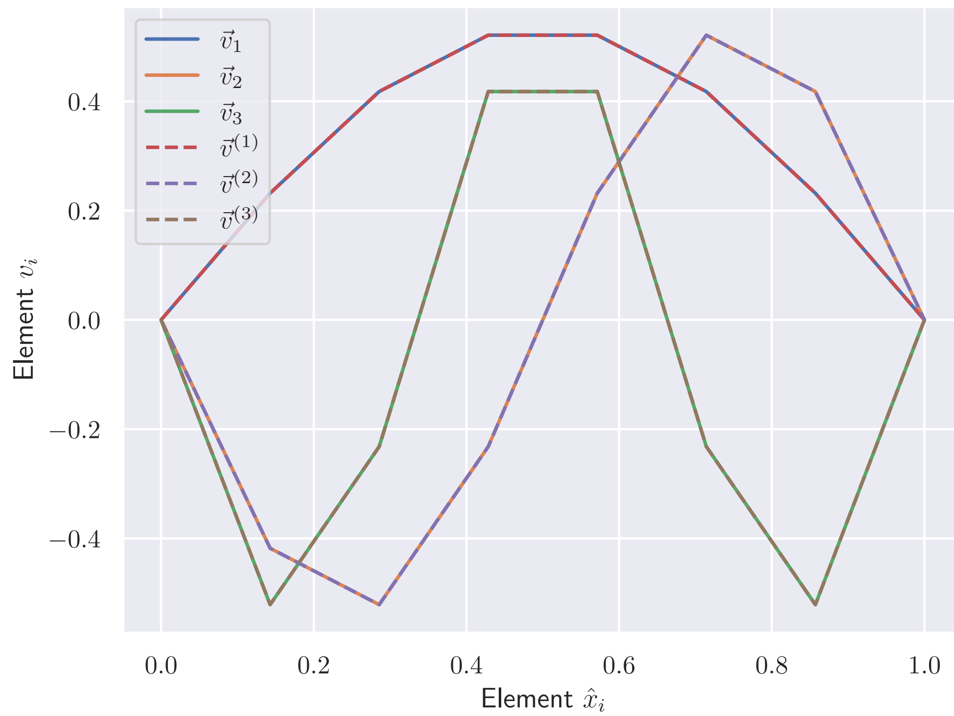

FIG. 2. The plot is showing the elements of eigenvector $\vec{v}_1, \vec{v}_2, \vec{v}_3$, corresponding to the three lowest eigenvalues of matrix $\boldsymbol{A}(6 \times 6)$, against the position $\hat{x}$. The analytical eigenvectors $\vec{v}^{(1)}, \vec{v}^{(2)}, \vec{v}^{(3)}$ are also included in the plot.

b)

For the discretization with $n = 100$ the solution is visually close to a continous curve, with a complete overlap of the analytical eigenvectors, presented in Figure 3.

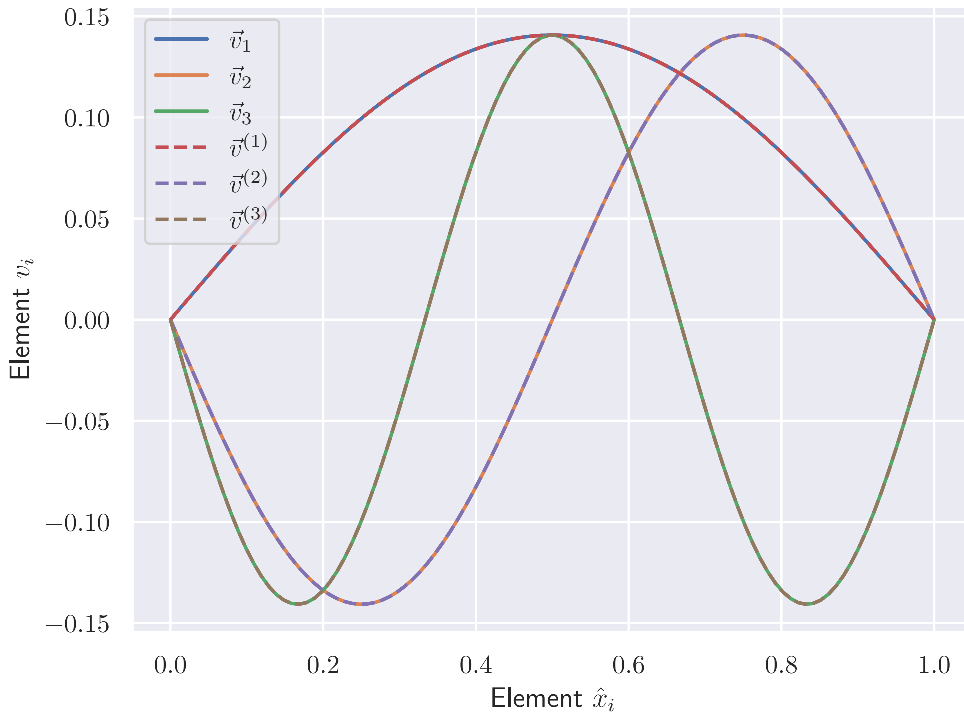

FIG. 3. The plot is showing the elements of eigenvector $\vec{v}_1, \vec{v}_2, \vec{v}_3$, corresponding to the three lowest eigenvalues of matrix $\boldsymbol{A}(100 \times 100)$, against the position $\hat{x}$. The analytical eigenvectors $\vec{v}^{(1)}, \vec{v}^{(2)}, \vec{v}^{(3)}$ are also included in the plot.