Project 3

Cory Alexander Balaton & Janita Ovidie Sandtrøen Willumsen
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Add an abstract for the project?

I. INTRODUCTION

We are surrounded by matter, which are made up of elementary particles. In the field of physics we want to understand the properties of these particles, measure their physical quantities, not to mention explain the origin of mass [1].

However, to study a particle, it is necessary isolate and contain it over time. The Penning trap is a device, able to confine charged particles for a period of time. This concept was evolved from F. M. Penning's implementation of magnetic fiels to a vaccum gauge, and J. R. Pierce's work with electron beams, and put into practice by Hans Dehmelt. In 1973 Dehmelt and his group of researchers were able to confine a particle and store it over several months [2].

In practice, a Penning trap is not easy to obtain, and an experiment is both time consuming and expensive. A numerical approach, allow us to study the effects of the Penning trap on a charged particle, without the cost. We can use ordinary differential equations to model the particle's movement, confined within an Penning trap. Our focus will be on an ideal Penning trap, where an electrostatic field confines the particle in z-direction, and a magnetic field confines it in the radial direction. We will use numerical methods to model a single particle, and study the particle motion in radial direction. In addition, we will model a system of particles, and study their motion both with and without particle interaction.

II. METHODS

When we study the Penning traps effect on a particle with a charge q, we need to consider the forces acting on the particle. The sum of all forces acting on the particle, is given by the Lorentz force (1).

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}, \tag{1}$$

We can use Newton's second law (7) to determine this sum by

$$\begin{aligned}\ddot{\mathbf{r}} &= \frac{1}{m}\sum_i \mathbf{F}_i \\ &= \frac{1}{m}(q\mathbf{E} + q\mathbf{v} \times \mathbf{B}) \\ &= \frac{q}{m}\big(\frac{V_0}{d^2}(x, y, -2z) + (B_0\dot{y}, -B_0\dot{x}, 0)\big),\end{aligned}$$

where the complete derivation can be found in . We can now write the particle's position as

$$\ddot{x} - \omega_0\dot{y} - \frac{1}{2}\omega_z^2 x = 0, \tag{2}$$

$$\ddot{y} + \omega_0\dot{x} - \frac{1}{2}\omega_z^2 y = 0, \tag{3}$$

$$\ddot{z} + \omega_z^2 z = 0, \tag{4}$$

In addition, we can find the general solution for eq. (4), when we consider the characteristic equation of a second order differential equation [3].

$$\begin{aligned}r^2 + \omega_z^2 &= 0 \\ r &= \pm\sqrt{-\omega_z^2} = \mp i\omega_z,\end{aligned}$$

two complex roots which gives us solutions in the form of

$$z = c_1 e^{i\omega_z t} + c_2 e^{-i\omega_z t}, \tag{5}$$

For a complex number $z = a + ib$, we can define $e^z \equiv e^a(\cos b + i\sin b)$ [4]. We can rewrite (5) as

$$\begin{aligned}c_1 e^{i\omega_z t} + c_2 e^{-i\omega_z t} &= c_1(\cos\omega_z t + i\sin\omega_z t) + c_2(\cos\omega_z t - i\sin\omega_z t \\ &= E\cos\omega_z t + iF\sin\omega_z t\end{aligned}$$

Since (2) and (3) are coupled, we want to rewrite it as a single differential equation. We can obtain this by introducing $f(t) = x(t) + iy(t)$,

$$\begin{aligned}(\ddot{x} - \omega_0\dot{y} - \frac{1}{2}\omega_z^2 x) + i(\ddot{y} + \omega_0\dot{x} - \frac{1}{2}\omega_z^2 y) &= 0 \\ \ddot{x} - \omega_0\dot{y} - \frac{1}{2}\omega_z^2 x + i\ddot{y} + i\omega_0\dot{x} - i\frac{1}{2}\omega_z^2 y &= 0 \\ \ddot{x} + i\ddot{y} + i\omega_0\dot{x} - \omega_0\dot{y} - \frac{1}{2}\omega_z^2 x - i\frac{1}{2}\omega_z^2 y &= 0 \\ \ddot{x} + i\ddot{y} + i\omega_0(\dot{x} + i\dot{y}) - \frac{1}{2}\omega_z^2 x - i\frac{1}{2}\omega_z^2 y &= 0 \text{where } i\omega_0\dot{x} + (-1)\omega_0\dot{y} = \\ \ddot{f} + i\omega_0\dot{f} - \frac{1}{2}\omega_z^2 f &= 0\end{aligned}$$

Physical properties given by newtons second law (7)

$$f(t) = A_+ e^{-i(\omega_+ t + \phi_+)} + A_- e^{-i(\omega_- t + \phi_-)} \tag{6}$$

The particle moves and its position can be determined using newton. where the electric field

Algorithm

Tools

We used matplotlib

III. RESULTS AND DISCUSSION

From equation (6) we can find

Problem 9: we have to deal with $|\mathbf{r}| = d$

IV. CONCLUSION

APPENDIX A

Equations given

$$m\ddot{\mathbf{r}} = \sum_i \mathbf{F}_i \tag{7}$$

$$\mathbf{E} = k_e \sum_{j=1}^{n} q_j \frac{\mathbf{r} - \mathbf{r}_j}{|\mathbf{r} - \mathbf{r}_j|^3} \tag{8}$$

$$\mathbf{E} = -\nabla V \tag{9}$$

APPENDIX B

Sum of all forces

$$sum$$

We find the physical coordinates from $x(t) = \mathrm{Re} f(t)$ and $y(t) = \mathrm{Im} f(t)$, where $f(t)$ is given by equation (6).

We can rewrite $f(t)$ using the definition as

$$\begin{aligned} f(t) =& A_+(\cos\omega_+ t + \phi_+ - i\sin\omega_+ t + \phi_+) \\ &+ A_-(\cos\omega_- t + \phi_- - i\sin\omega_- t + \phi_-) \end{aligned} \tag{10}$$

If we rearrange the right side of equation (10), we find the physical coordinates

$$x(t) = A_+(\cos\omega_+ t + \phi_+) + A_-(\cos\omega_- t + \phi_-) \tag{11}$$

$$y(t) = -A_+(i\sin\omega_+ t + \phi_+) - A_-(i\sin\omega_- t + \phi_-) \tag{12}$$

[1] The Editors of Encyclopaedia Britannica. matter. 2023.

[2] Manuel Vogel. *Particle Confinement in Penning Traps*. Springer Cham, 1–6 edition, 2018.

[3] Tom Lindstrøm. *Kalkulus*, pages 589–606. Universitetsforlaget, 4 edition, 2016.

[4] Tom Lindstrøm. *Kalkulus*, pages 133–137. Universitetsforlaget, 4 edition, 2016.